BJT Small-Signal Equivalent Circuit Representation

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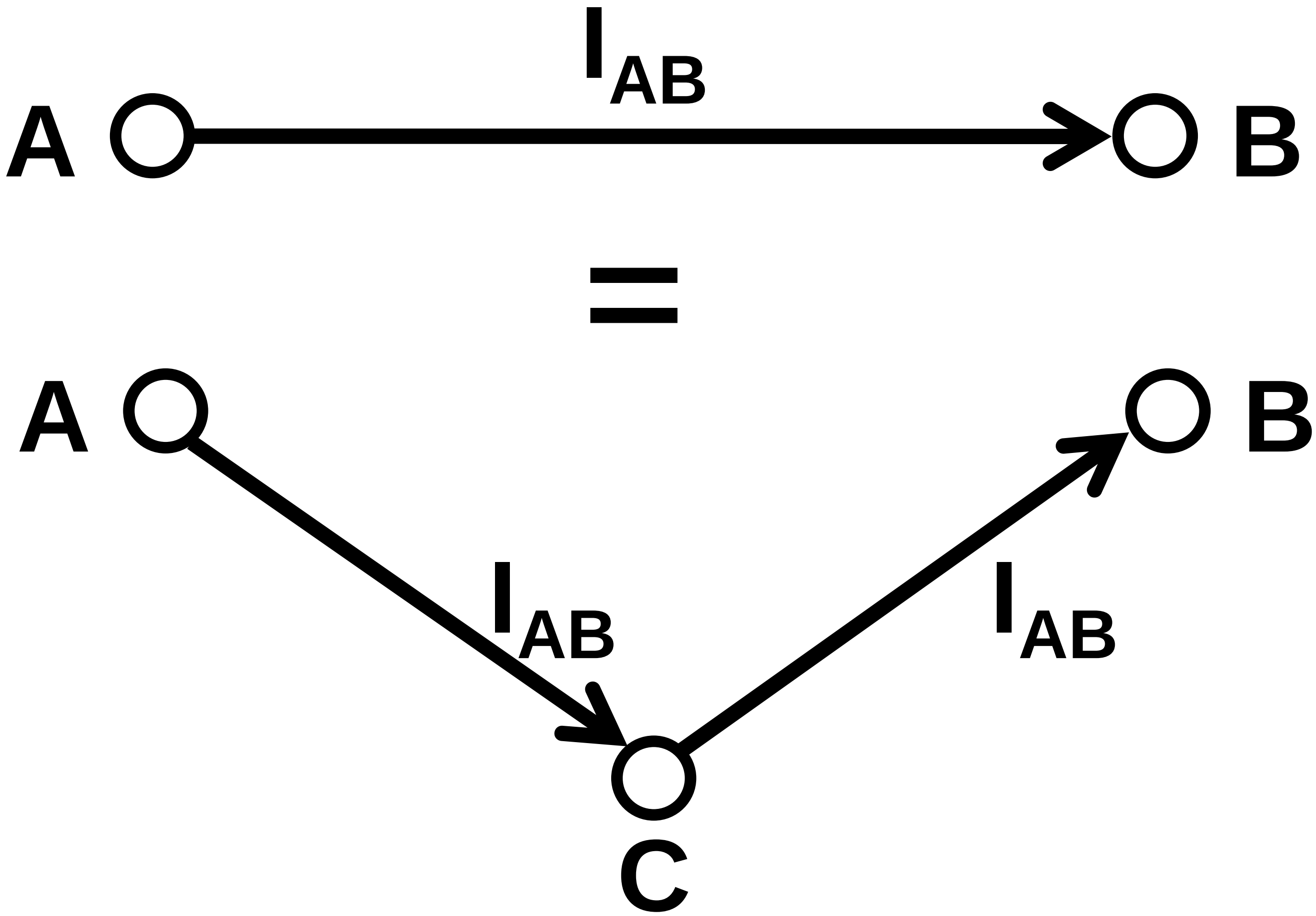
IEEE BCTM Paper 9.1
October 5, 2010

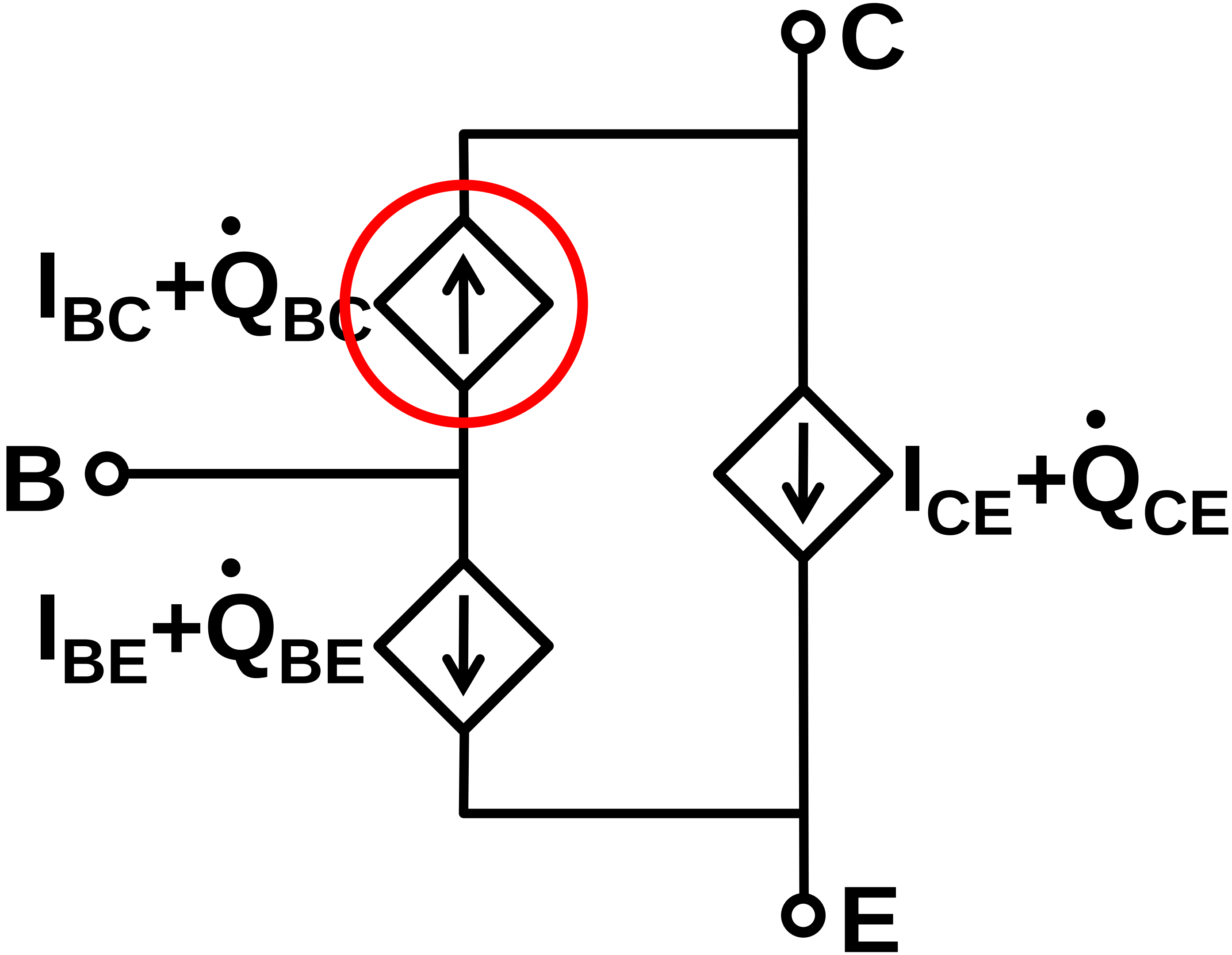
Overview

- Large-Signal Equivalent Circuit for BJTs
- Linearization for Small-Signal Modeling
  - number and topology of branches in small-signal model
  - standard hybrid-π representation
  - missing capacitance elements
  - relation to MOS small-signal model
- Characterization from Data
- Summary

Current Transformation

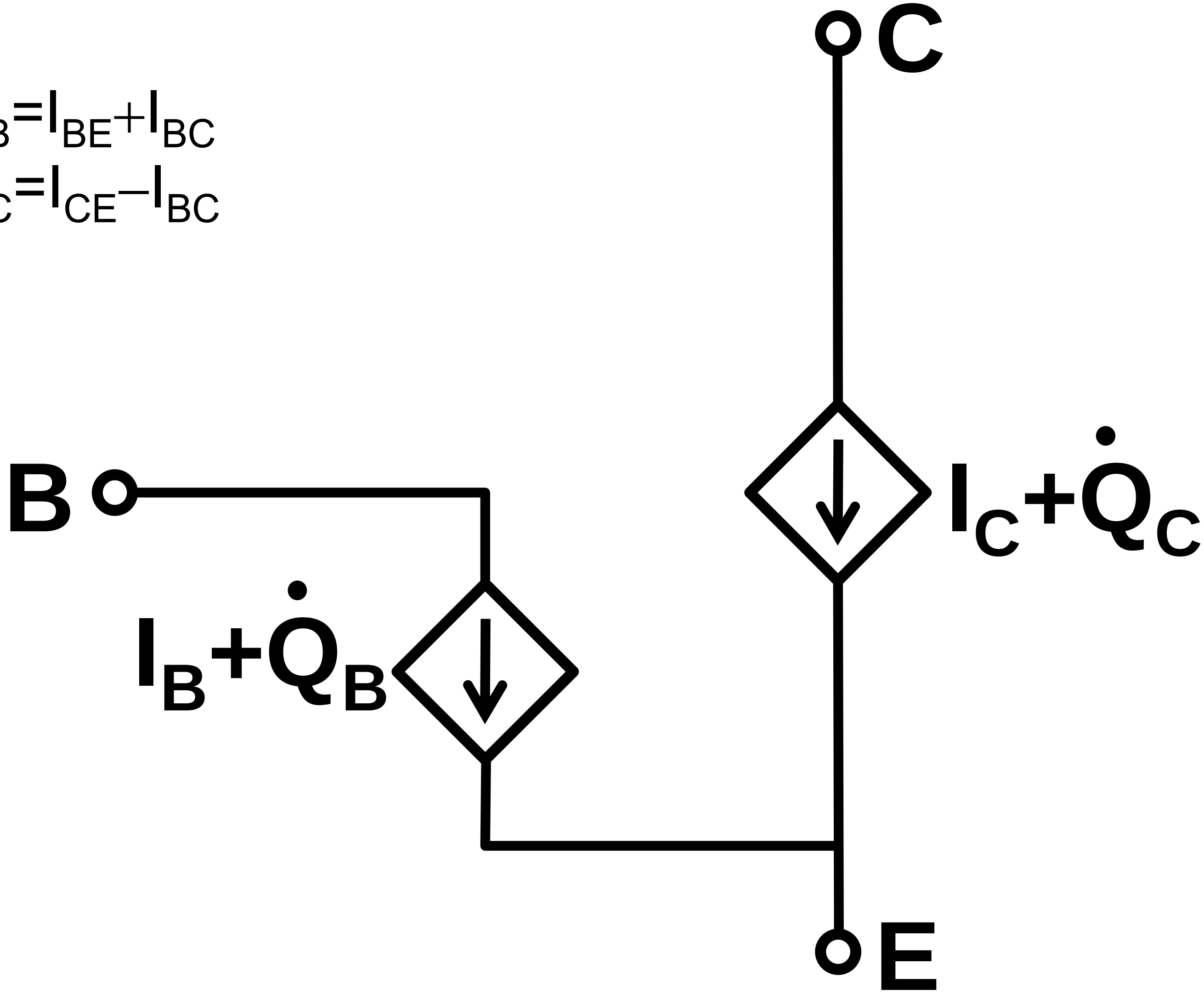


Large-Signal Model (3-Terminal BJT, Intrinsic)



Large-Signal Model (3-Terminal BJT, Intrinsic)

$I_B = I_{BE} + I_{BC}$
$I_C = I_{CE} - I_{BC}$



General Large-Signal Model

- KCL for transport (dc) currents: $I_E + I_B + I_C = 0$
- Charge neutrality: $Q_E + Q_B + Q_C = 0$
- KCL for charging (capacitive) currents: $\dot{Q}_E + \dot{Q}_B + \dot{Q}_C = 0$

- Base current: $I_B = I_{BE}(V_{BE}, V_{BC}) + I_{BC}(V_{BC}, V_{BE})$
  - neutral-base recombination, impact ionization

- Gummel integral charge control: $I_{CE} = I_{CE}(V_{BE}, V_{BC})$

- Charges: $Q_{BE}(V_{BE}, V_{BC})$, $Q_{BC}(V_{BC}, V_{BE})$
  - junctions charges $Q_{j,BE}(V_{BE})$, $Q_{j,BC}(V_{BC})$
  - transport charge “cross” dependence
    - > charge partitioning

Small-Signal Model

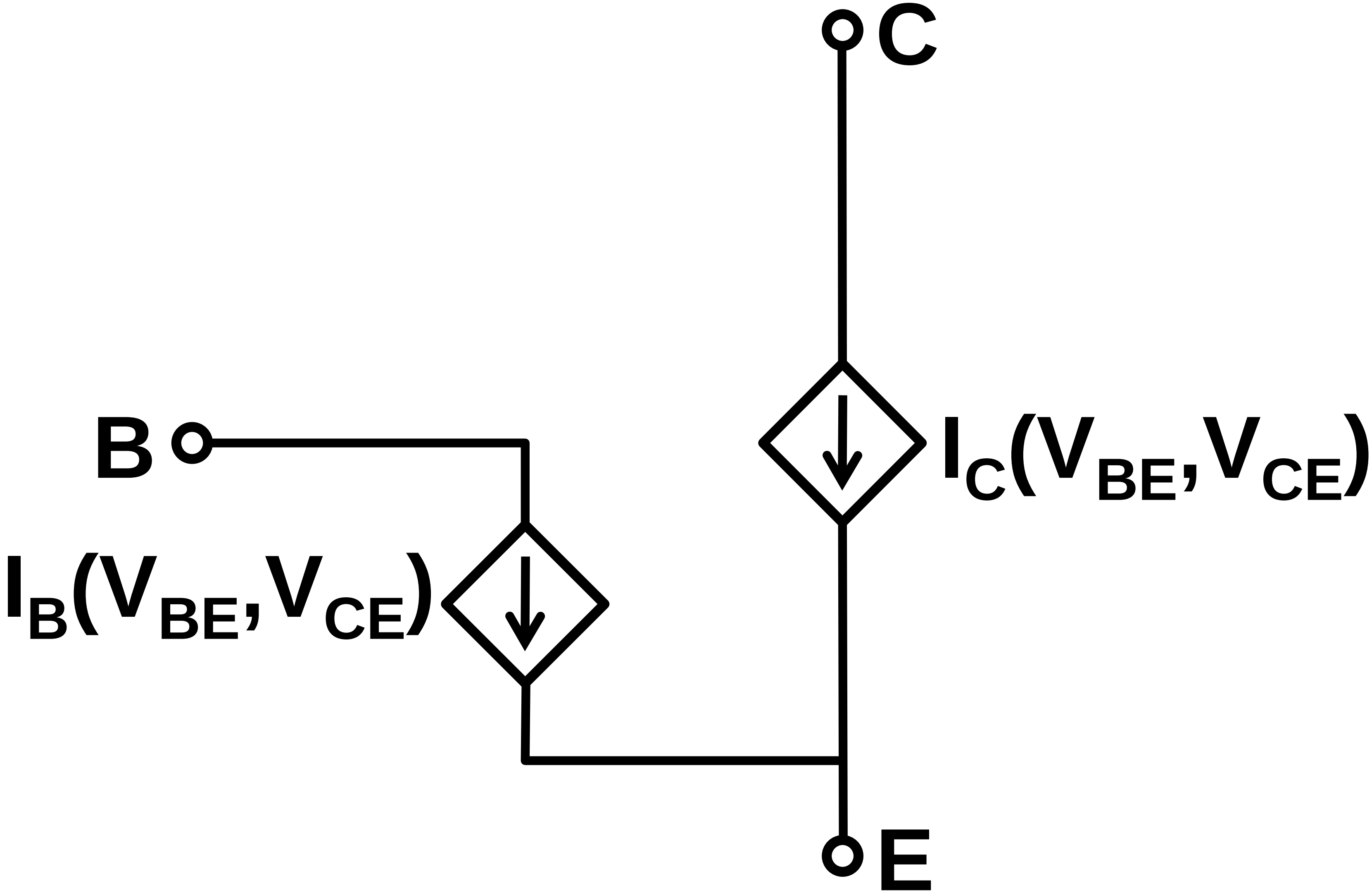
- Generic two-port y-parameter model

$$\begin{bmatrix} I_b \\ I_c \end{bmatrix} = \begin{bmatrix} y_{bb} & y_{bc} \\ y_{cb} & y_{cc} \end{bmatrix} \begin{bmatrix} V_b \\ V_c \end{bmatrix}$$

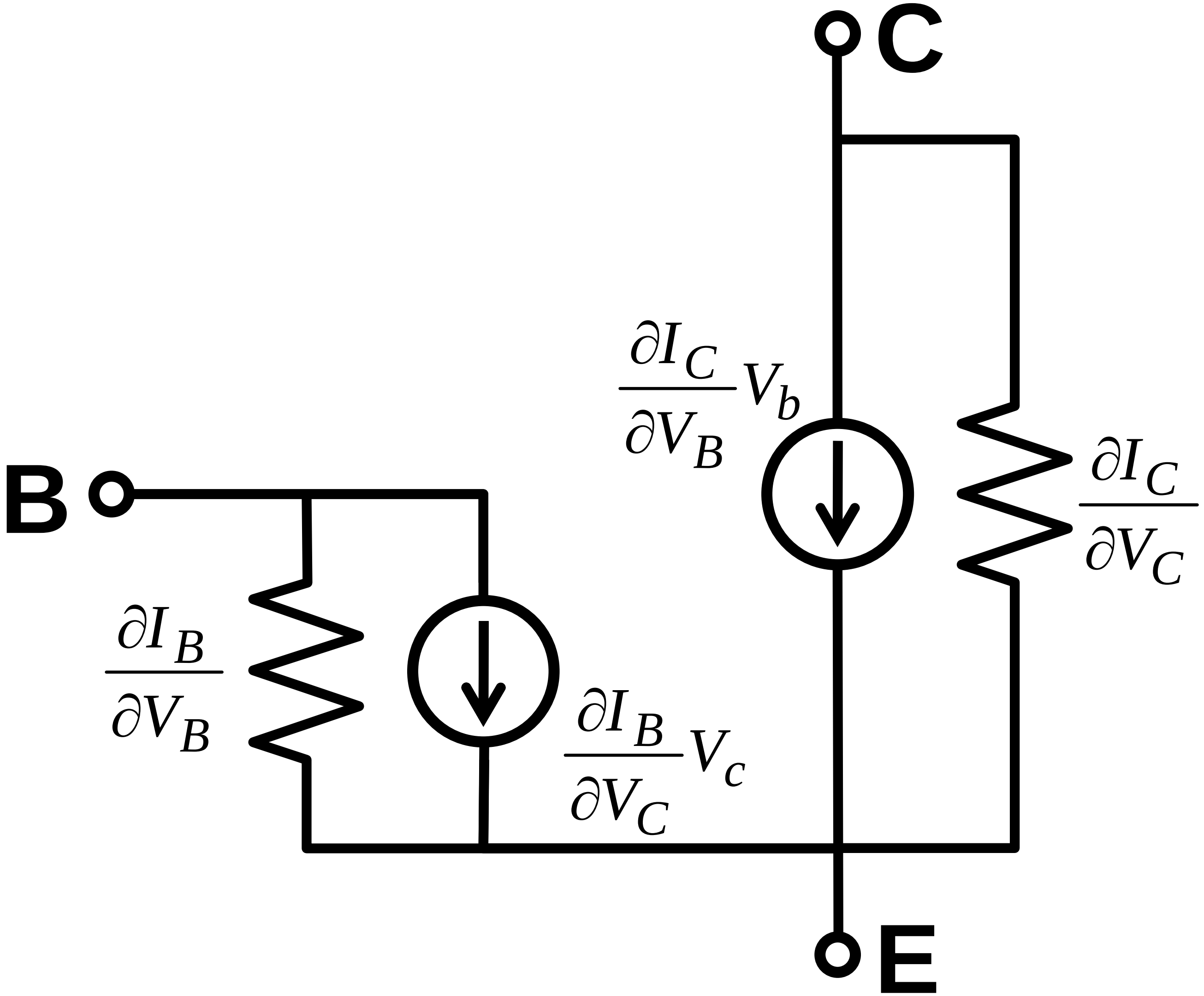
❖ **4 elements**

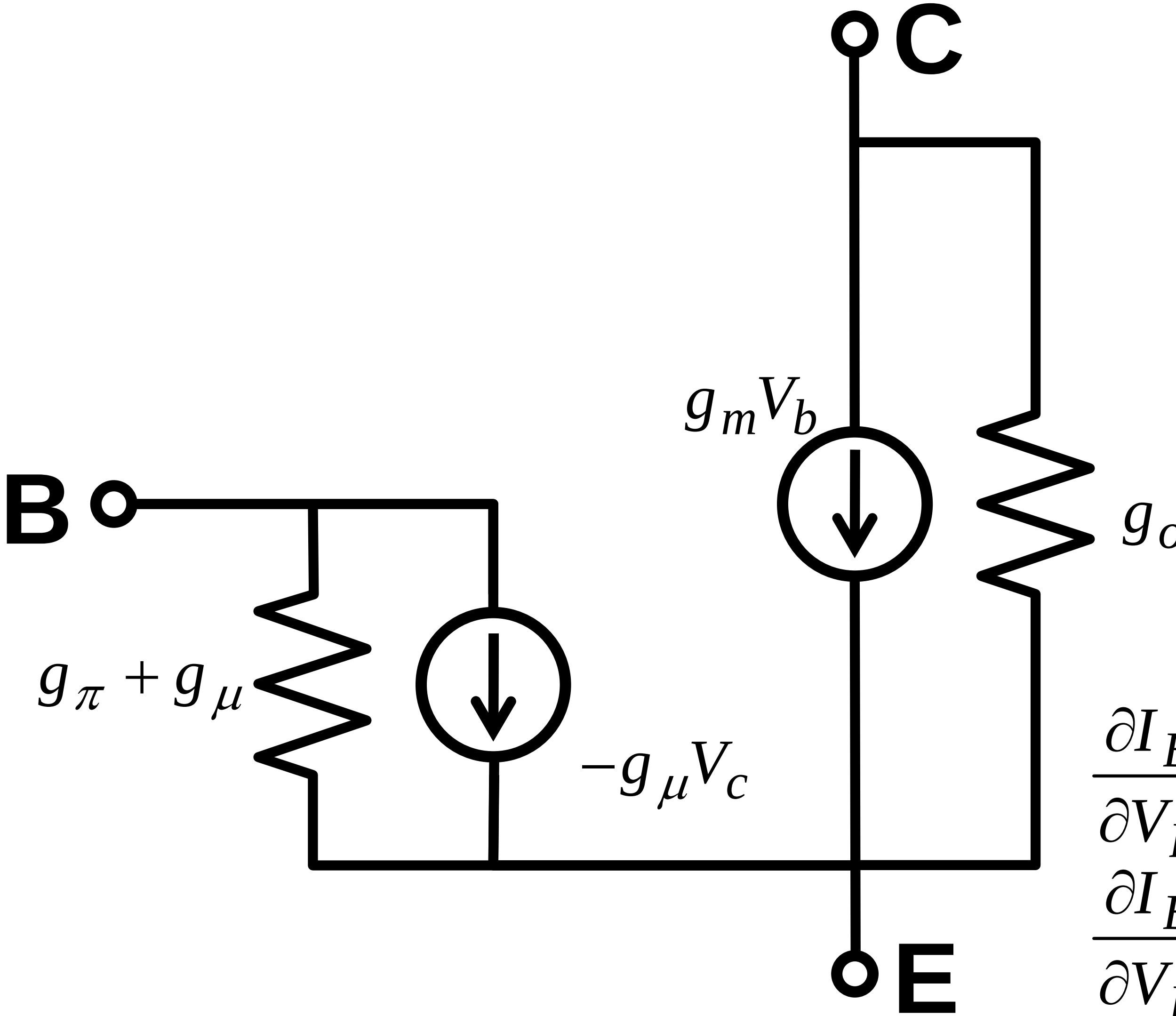
$$= \begin{bmatrix} g_{bb} + j\omega C_{bb} & g_{bc} + j\omega C_{bc} \\ g_{cb} + j\omega C_{cb} & g_{cc} + j\omega C_{cc} \end{bmatrix} \begin{bmatrix} V_b \\ V_c \end{bmatrix}$$

Large-Signal Model (3-Terminal BJT, Intrinsic I)



Low Frequency Small-Signal Model

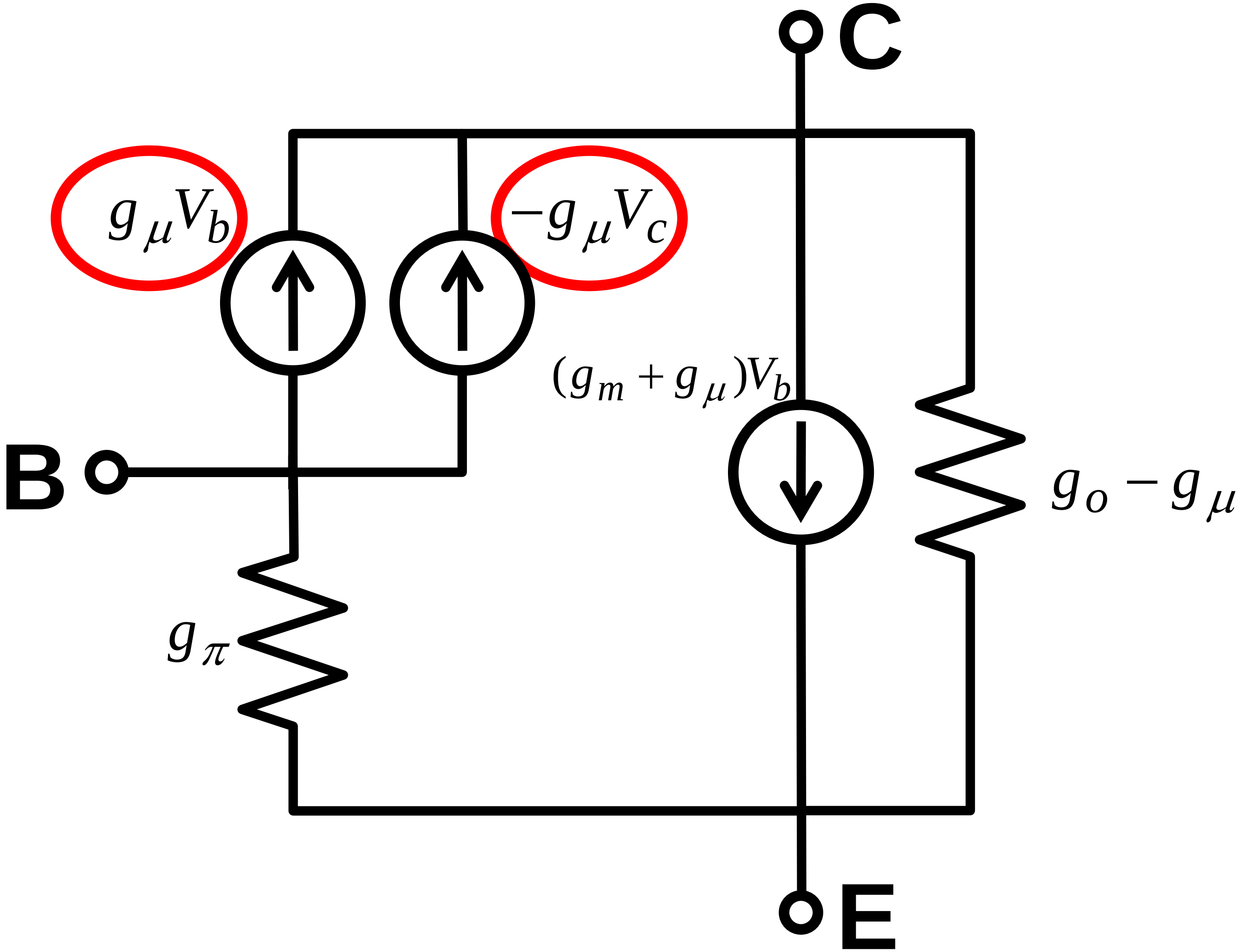




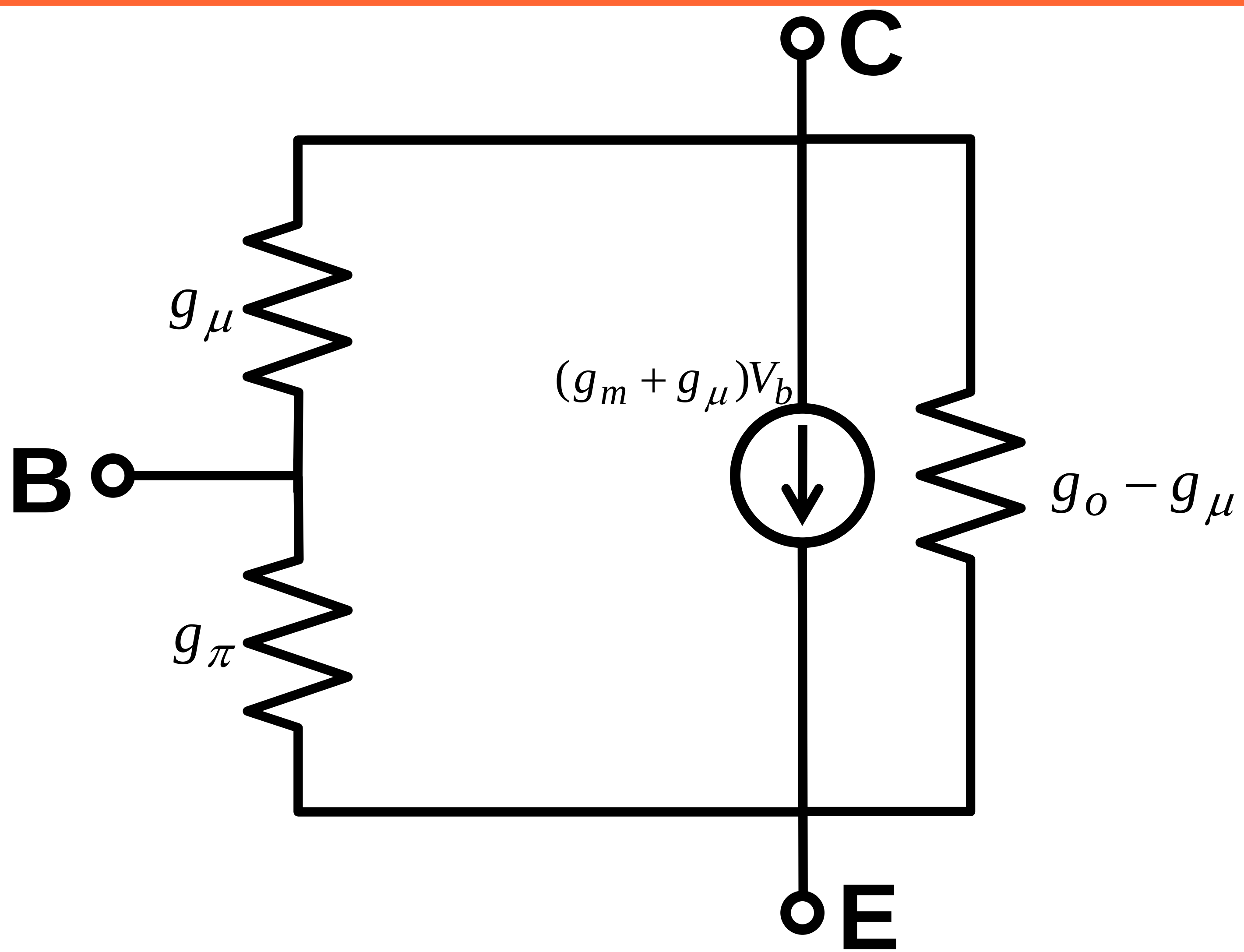
$$\frac{\partial I_B}{\partial V_B} + \frac{\partial I_B}{\partial V_E} + \frac{\partial I_B}{\partial V_C} = 0$$

$$\frac{\partial I_B}{\partial V_B} - g_\pi - g_\mu = 0$$

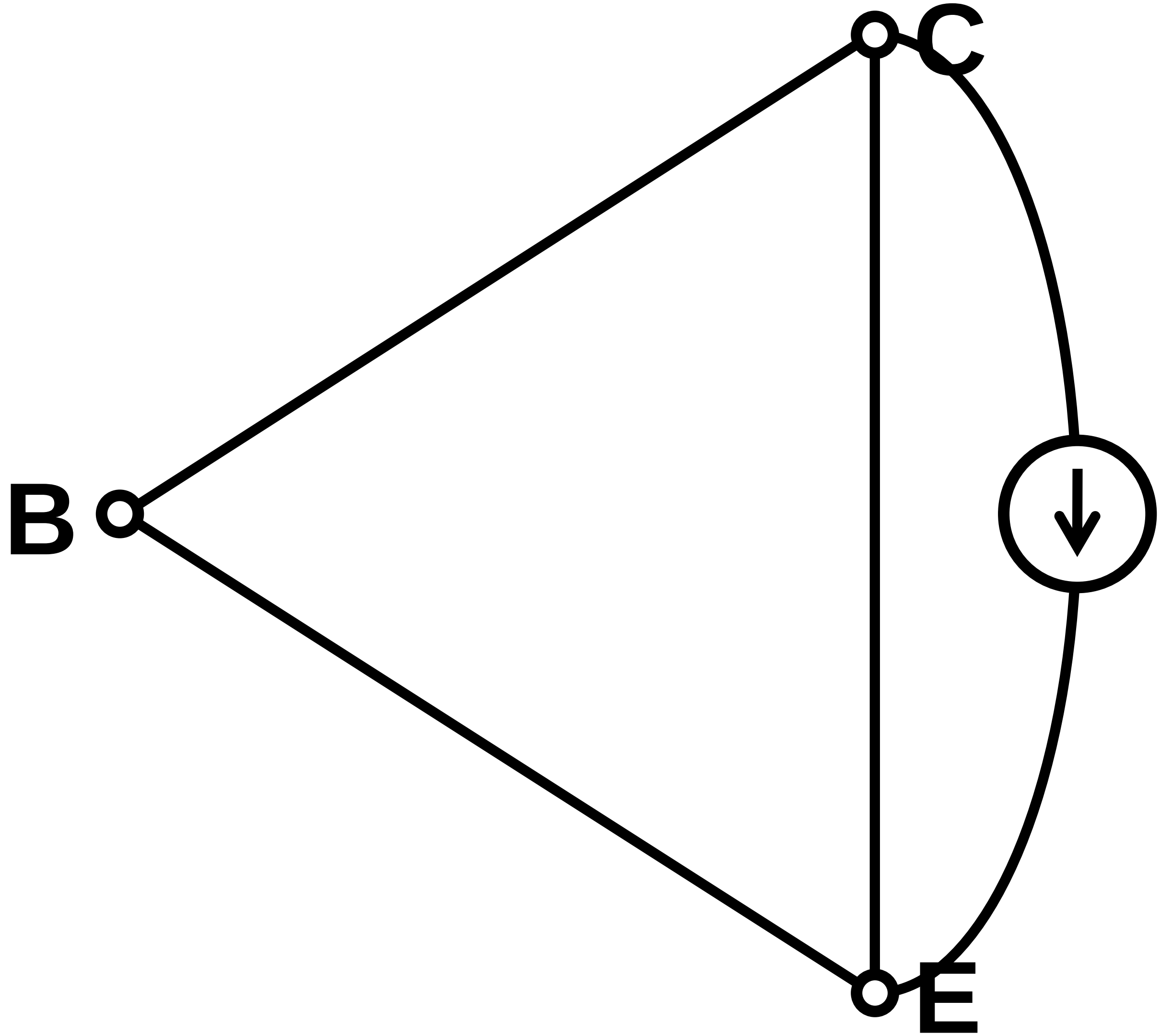
Low Frequency Small-Signal Model



Low Frequency Small-Signal Model

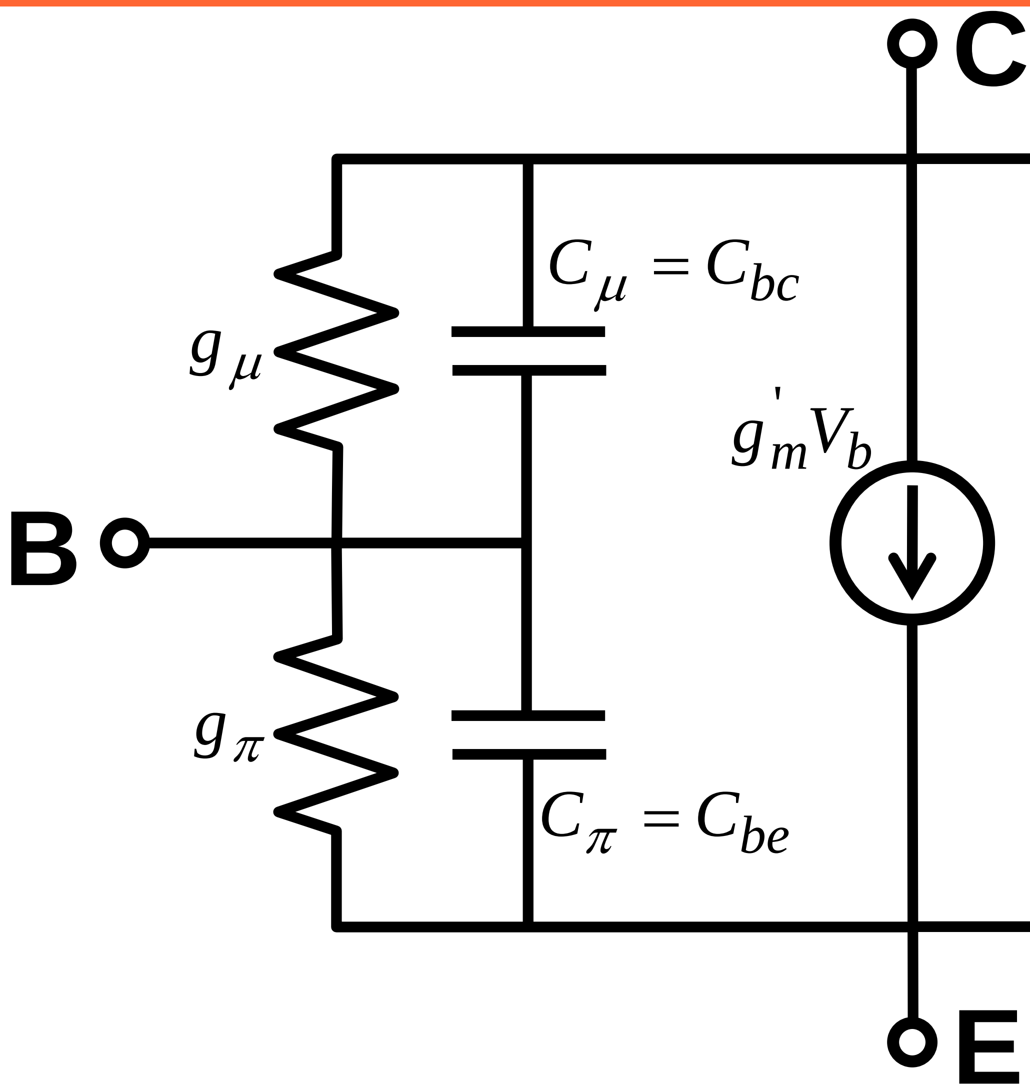


Small-Signal Model Topology



- only 3 "direct connections"
- topologically ≥1 "trans" element
- both for real (g) and imaginary (C) parts

Conventional Hybrid-π Small-Signal Model



$$C_{mf} = -\frac{\partial Q_M}{\partial V_F}$$

What's Wrong with this Hybrid-π Model?

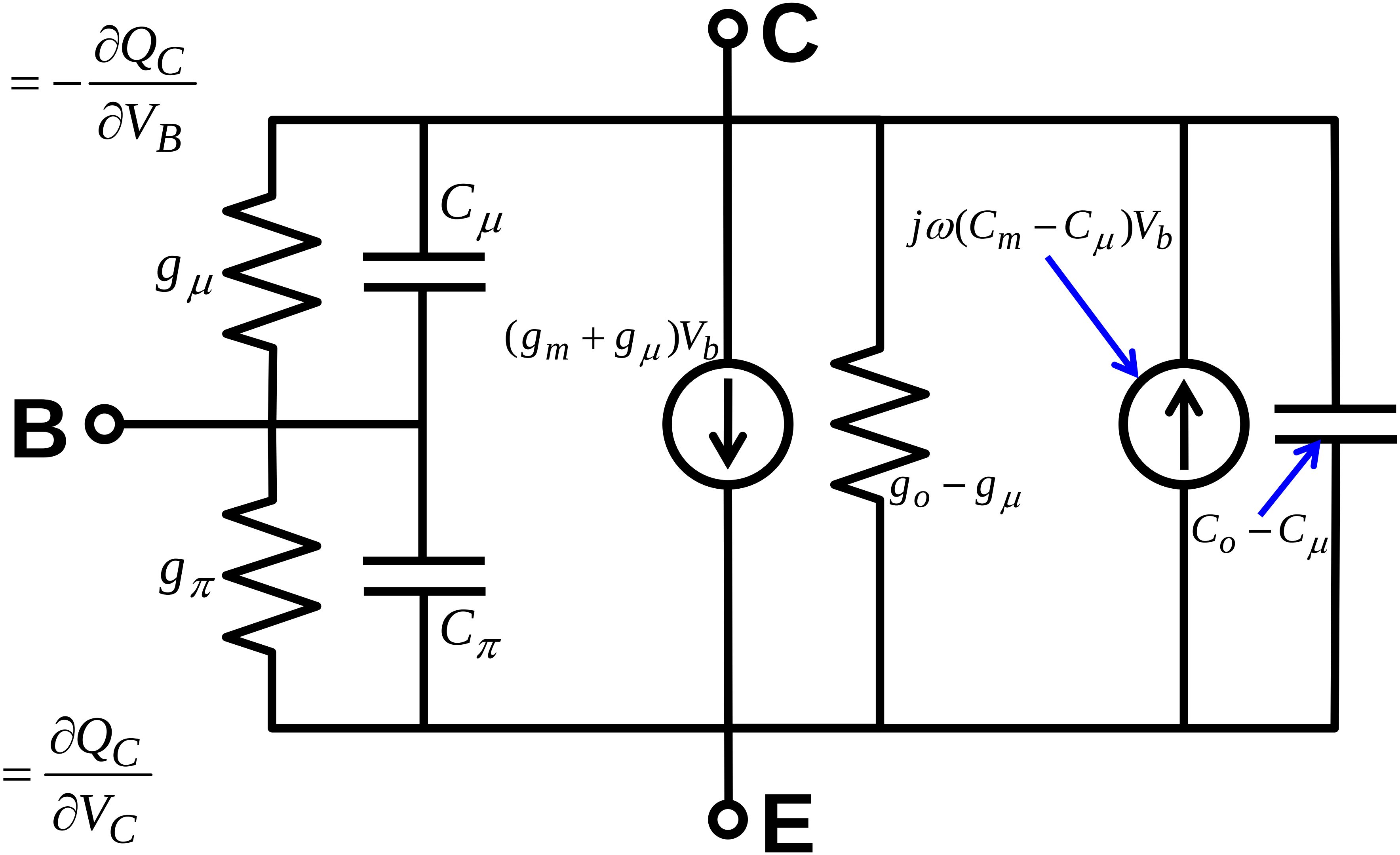
- There are only two capacitive components
  - topologically there must be four, two are completely missing!
  - at least one of which must be a transcapacitance
- The conventional transconductance and output conductance terms do not represent
  - intuitive expectation of what these components mean
  - how they are commonly determined from measurement

$$g_m = \frac{\partial I_C}{\partial V_B} \neq \frac{\partial I_C}{\partial V_B} - \frac{\partial I_B}{\partial V_C} = g_m'$$

$$g_o = \frac{\partial I_C}{\partial V_C} \neq \frac{\partial I_C}{\partial V_C} + \frac{\partial I_B}{\partial V_C} = g_o'$$

Exact and Complete Small-Signal Model

$$C_m = -\frac{\partial Q_C}{\partial V_B}$$



$$C_o = \frac{\partial Q_C}{\partial V_C}$$

Relations Between Capacitances

$$C_o - C_\mu = C_{cc} - C_{be} = C_{ec} = -\frac{\partial Q_E}{\partial V_C}$$

$$C_m - C_\mu = C_{cb} - C_{bc} = -\frac{\partial Q_C}{\partial V_B} + \frac{\partial Q_B}{\partial V_C}$$

equivalent of Tsividis C_m for MOS

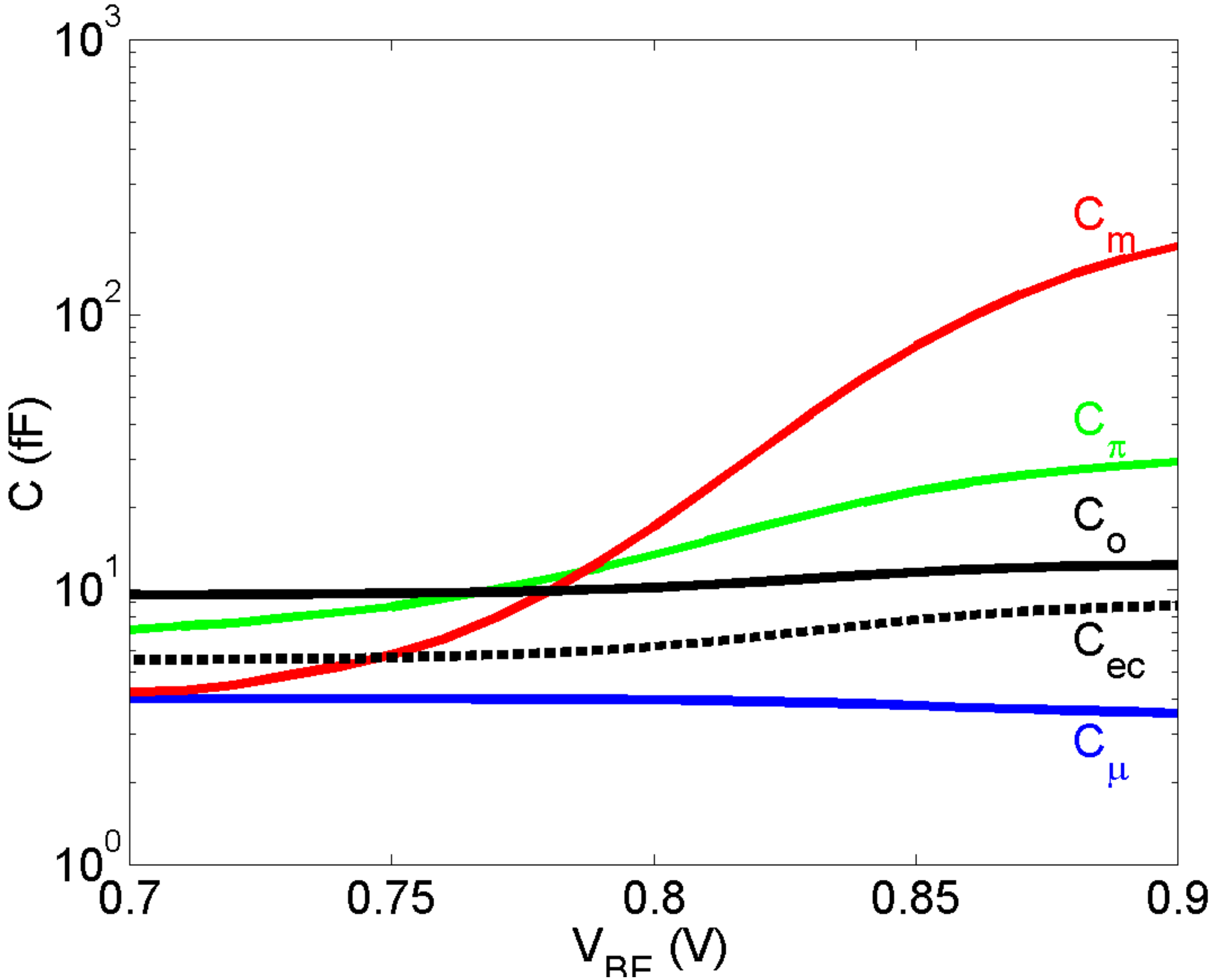
Properties

- Low-level injection (transport charge is negligible)
  - Q_E depends only on V_{BE}, $C_{ec}=0$ hence $C_o-C_\mu=0$
  - base-collector capacitance is reciprocal, $C_{cb}=C_{bc}$, hence $C_m-C_\mu=0$
  - conventional hybrid-π representation is fine
- Added capacitance elements can be significant when transport charges becomes appreciable
  - Q_E depends on V_{CE}
  - "cross dependence" of terminal charges on biases is affected by partitioning of the transport charge

SiGe HBT Data, V_{CE}=2.0 V, f=30GHz

low frequency:

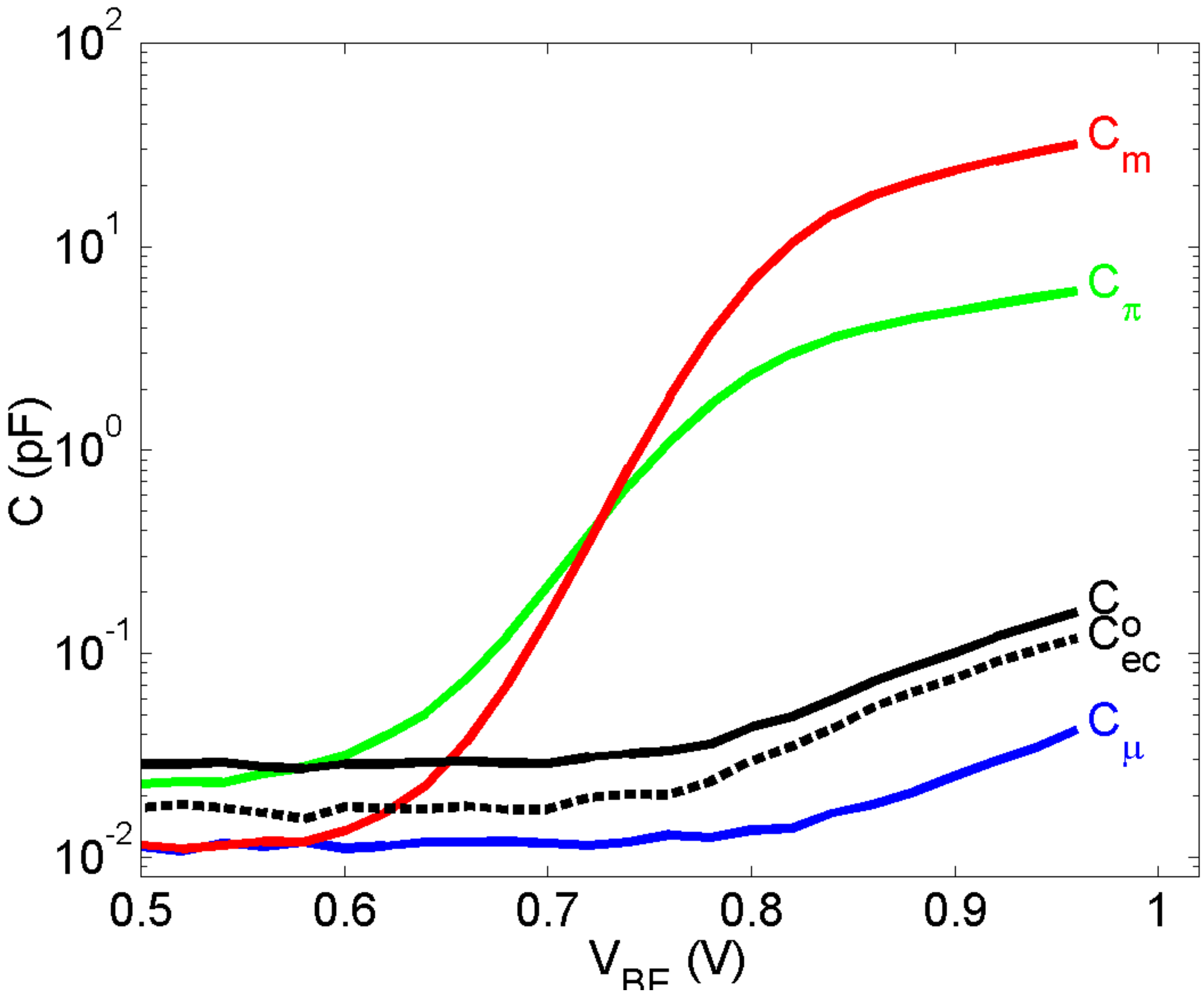
$C_m = C_\mu = C_o - C_{sc}$



Si BJT Data, V_{CE}=10.0 V, f=100MHz

low frequency:

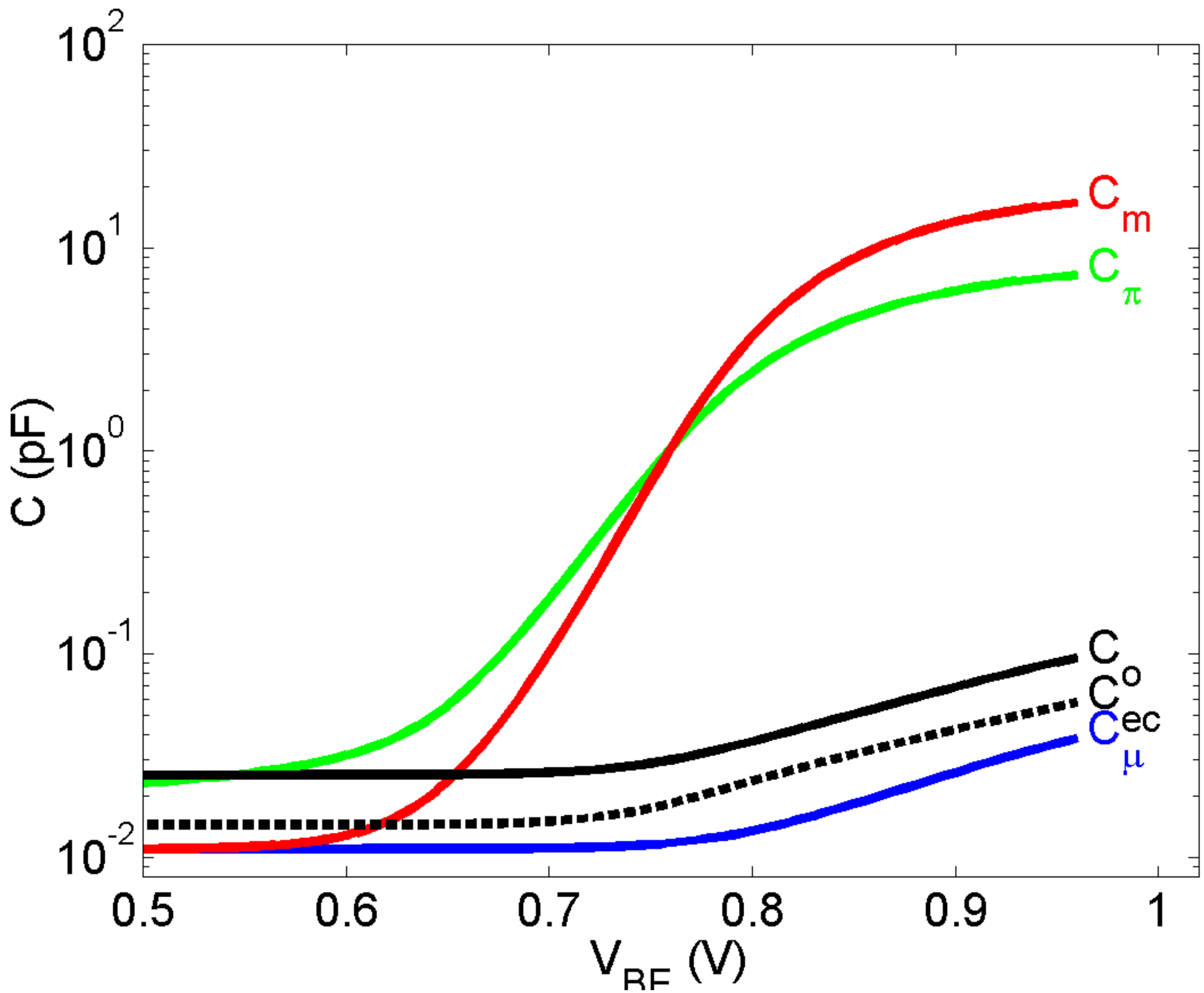
$C_m = C_\mu = C_o - C_{sc}$



Si BJT VBIC Simulation, V_{CE}=10.0 V, f=100MHz

low frequency:

$C_m = C_\mu = C_o - C_{sc}$



Observations

- Simulator implementations of BJT models are ***not*** wrong
  - linearization used for dc convergence and ac modeling is fine
  - guaranteed correct for Verilog-A defined models
- Representation via operating point analysis is incomplete and therefore inaccurate
  - design decisions based on this can be wrong
  - simulator operating point representation is ***always*** misleading
    - > terminal small-signal parameters like g_m differ from their intrinsic device values due to parasitics anyway (e.g. $g_m \rightarrow g_m/(1+g_m R_E)$)
- f_T estimation is ***not*** affected by imprecision in small-signal hybrid-π model
  - $f_T = g_m/C_{bb}$
  - g_m modeling inaccuracy usually insignificant (except avalanche)
  - C_{bb} modeling independent of C_m and C_o

Conclusions

- Hybrid-π BJT small-signal model representation used for more than 50 years is incomplete and imprecise
  - two capacitance components are missing
    - > one is a transcapacitance
  - representations of g_m and g_o are inconsistent with how they are commonly determined from measurement
    - > should be based on terminal, not branch, currents
- Presented a complete small-signal model for the intrinsic 3-terminal bipolar transistor
  - has a minimum number (1) of “trans” elements
  - has intuitive definitions for g_m and g_o
- Investigation of C_m should help elucidate partitioning of transport charge

Acknowledgements

- The model presented here is directly derived from the MOS small-signal model from “*Operation and Modeling of the MOS Transistor*” by Prof. Yannis Tsividis (1987)
  - his representation is exact for any 4-terminal device
  - simplified here for arbitrary 3-terminal device
  - somewhat ironic that MOS (generic) solution did not propagate to BJT for 23 years ...